SECOND SEMESTER B.Sc. DEGREE EXAMINATION
MATHEMATICS (COMPLEMENTARY COURSE)
MM2C02: MATHEMATICS
Model Question Paper 2

Time: 3 hrs. Maximum Weightage: 30

I. Objective type questions (Answer all 12 questions, wieghtage 12 x ¼ = 3)

Fill in the blanks

1. The value of k such that the integral $\int_1^\infty x^{-k}dx$ diverges is ....
2. An example of a divergent sequence ....
3. A convergent subsequence of the sequence $\left\{\left(\sin n\frac{\pi}{2}\right)\right\}$......................
4. The n^{th} term of the sequence $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \ldots$ is ....
5. The series $\sum_{n=1}^{\infty} n^2$ is ....
6. The derivative of $y = \operatorname{cosec} h(3x)$ w.r. to x is ....
7. The value of $\sin h^{-1} 1$ using logarithm is ....
8. The range of $\cot h^{-1} x$ is ....
9. The cylindrical coordinates of $(0,1,0)$ (in Cartesian coordinate) is ....
10. In polar coordinates the region $\{(x, y) : x \geq 0, y \geq 0\}$ (in cartesian coordinates) is ....
11. The graph of the polar equation $r = 2$ is a ....
12. If $z = e^{xy}.\ln y$ then $\frac{\partial z}{\partial x} =$....................

II. Short answer type questions (Answer all 9 questions, weightage 9x1 = 9)

13. Find $\int_0^1 \frac{dx}{\sqrt{x}}$.
14. Show that $\lim_{n\to\infty} \frac{\ln}{n} = 0$.
15. Check the convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$
16. Show that the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges.

17. State the series multiplication theorem for power series.
18. Define boundary point. Give an example.
19. Define a function $f(x, y)$ is continuous at the point (x_0, y_0).
20. Find the gradient of $f(x, y) = y - x^2$ at $(-1,0)$.
21. Find $\frac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$; $t = \pi$.

III. Short Essay questions (Answer any 5 questions, weightage 5 x 2 = 10)

22. Investigate the convergence of $\int_1^{\infty} e^{-x^2} dx$.
23. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2} \right)$.
24. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$. Where, if anywhere, does the series converge to $\frac{1}{x}$?
25. Graph the curve $r^2 = \sin 2\theta$
26. Find f_x, f_y and f_z for the function $f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$.
27. Find the linearizations of the function $f(x, y, z) = e^x + \cos(y + z)$ at $(0,0,0)$.
28. Find the equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point $(-2,1)$.

IV. Essay questions (Answer any 2 questions, weightage 2x4 = 8)

29. a. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} n^{-n}$.

 b. Show that the alteration harmonic series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges.

 c. Define absolute convergence and conditional convergence of a series what is the relation between these two?

30.(i) a. Find the points of intersection of the curves $r = 1 + \cos\theta$, $r^2 = 4\cos\theta$.

 b. Graph the curve $r = \frac{8}{4 + \cos\theta}$.

 c. Find the area of the region shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$.

or

31. a. Find $\dfrac{\partial w}{\partial v}$ when $u = 0$, $v = 0$ if $w = x^2 + \left(\dfrac{y}{x}\right)$, $x = u - 2v + 1$, $y = 2u + v - 2$.

b. Find the directional derivative of $f(x, y) = 2xy - 3y^2$ at $(5,5)$ in the direction $\vec{a} = 4\vec{i} + 3\vec{j}$.

c. Find the equation for the tangent plane and the normal line for the surface $x^2 + y^2 + z^2 = 3$ at $(1,1)$.