Model Question Paper

MM2C02 COMPLEMENTARY MATHEMATICS

Time : 3 Hours Maximum : 30 Weights

Part A : (3 Bunches of Objective Type Questions.)
(Answer All Questions.)

3 X 1 = 3 Weights.

1. Expression for cosh x in terms of tanh x is $\cdots$.
2. Cartesian equation corresponding to the polar equation $r = -1$ is $\cdots$.
3. Polar equation of a straight line passing through the pole and making an angle of 30^0 with the initial ray is $\cdots$.
4. $\int_1^\infty \frac{dx}{x^2} = \cdots$
5. Say True or False "The set of points in the first quadrant is a bouhnded set in the plane".
6. Radius of the circle $r = 6\cos\theta$ is $\cdots$.
7. $\lim_{n\to\infty}(1 - \frac{1}{2n})^n = \cdots$.
8. $\lim_{(x,y)\to(0,0)} \frac{e^{xy}\sin x}{x} = \cdots$.
9. The n^{th} term of the sequence $3, 2, \frac{5}{3}, \frac{6}{4}, \frac{7}{5}, \cdots$ is $\cdots$.
10. The least upper bound for the sequence $2, \frac{7}{3}, \frac{10}{4}, \frac{13}{5}, \frac{16}{6}, \cdots$ is $\cdots$.
11. Sum of the series $\Sigma_{n=1}^{\infty} \frac{(-1)^n 5}{4^n} = \cdots$.
12. The radius of convergence of the series $\Sigma_{n=1}^{\infty} \frac{3^n x^n}{n!}$ is $\cdots$.

Part B : (Very Short Answer Type Questions.)
(Answer All Questions.)

9 X 1 = 9 Weights.

13. Differentiate $\tanh\sqrt{1+x^2}$ with respect to x.

14. Express the equation $z^2 = r^3$ in the spherical polar form.

15. Prove or disprove : "Bounded sequences are always convergent".

16. test for convergence the sequence whose nth term is $\frac{1+\sqrt{2n}}{\sqrt{n}}$.

17. Find $\lim_{n\to\infty} \frac{\ln(n^2)}{n}$.

18. Find the domain and range of the function $f(x, y) = \sqrt{x^2 - y}$.

19. Express the repeating decimal $0.2\overline{34}$ as a ratio of two integers.

20. If f(x,y,z) = $\sin x + e^{xy} + z$, then find the gradiehnt of f(x,y,z).

21. Expand e^x in ascending powers of x–a.

Part C : (Short Essay Type Questions.)
(Answer Any Five Questions.)

5 X 2 = 10 Weights.

22. Show that $\int_1^\infty \frac{3}{e^x+5}$ converges.

23. If x is real, show that $\sinh^{-1}$x = log(x+$\sqrt{x^3+1}$).

24. Find the angle between the lines passing through (r_1, θ_1) and (r_2, θ_2). Deduce the condition for the lines to be perpendicular.

25. Find the area inside the circle r = 3 a $\cos\theta$ and
outside the cardioid r = a (1+$\cos\theta$).

26. If V = $\frac{1}{r}$ and $r^2 = x^2 + y^2$, show that

$$\frac{\delta^2 V}{\delta^2 x} + \frac{\delta^2 V}{\delta^2 y} = \frac{1}{r^3}.$$

27. Determine the interval of convergence and the radius of convergence for the power series $\Sigma_{n=1}^{\infty} \frac{(-1)^n (n+1)(x+1)^n}{2^n}$.

28. Does the series $\Sigma_{n=2}^{\infty} (\frac{n+1}{n-1})^n$ converge ?

Part D : (Esay Type Questions.)
(Answer Any Two Questions.)

2 X 4 = 8 Weights.

29. (a) Find the area of the surface generated by revolving the right hand loop of the lemniscate $r^2 = \cos 2\theta$ about the y-axis.

    (b) Discuss thne continuity of the function

$$f(x,y) = \begin{cases} \frac{4x^2y}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$$

30. (a) Discuss the convergence (both Conditional and Absolute) of the series $\Sigma_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$.

    (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2). Also equations of the tangent lines to these surfaces at the origin.

31. (a) If V = f($\frac{x}{z}, \frac{y}{z}$), show that $x\frac{\delta V}{\delta x} + y\frac{\delta V}{\delta y} + z\frac{\delta V}{\delta z} = 0$.

    (b) Using the Maclaurin's series, expand tan x upto the term containing x^5.