SECOND SEMESTER B.Sc. DEGREE EXAMINATION
MATHEMATICS (COMPLEMENTARY COURSE)

MM2C02: MATHEMATICS

Model Question Paper 1

Time: 3 hrs ***Maximum weightage : 30***

I. Objective type questions (Answer all questions, Weightage 12x¼ = 3)

1. The range of the function $y = \cos hx$ is ………………..
2. An example of an increasing sequence which is bounded above is ………………
3. The derivative of $\sec h2x$ with respect to x is ……………..
4. A convergent subsequence of the sequence $\{(-1)^n\}$ is ……………….
5. The n[th] term of the sequence 0,1,2,2,3,3,4,4, is ……………….
6. $\lim_{n\to\infty} a_n$ where $a_n = \sqrt{\dfrac{3n}{n+3}}$ is
7. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n$ = ……………
8. The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$ is …………
9. The polar equation of the cartesian equation $x = 7$ is……………
10. The radius of the circle $r = 6 \sin \theta$ is …………….
11. An example of a bounded region in the plane is ………………
12. Cartesian coordinates of the point (1,0) (given in polar coordinates) is …………………

II. Short answer type questions (Answer all nine questions. Weightage 9 x 1 = 9).

13. $\displaystyle\int_{-\ln 4}^{-\ln 2} 2e^x \cos hx \, dx$
14. Evaluate $\int_{-1}^{1} \frac{dx}{x^{2/3}}$.
15. Fin $\lim_{n\to\infty} a_n$ where $a_n = \dfrac{\sin n}{n}$.
16. Use partial fractions to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$.

17 Examine the convergence of the series $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n$. If it converges find its sum.

18 Prove or disprove $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ converges.

19 State Rearrangement theorem for absolutely convergent series.

20 Find an equation of the hyperbola with eccentricity $\frac{3}{2}$ and directix $x=2$.

21. Find the gradient of $f(x,y)=y-x$ at $(2,1)$.

III. Short essay questions (Answer any five questions, weightage 5 x 2 = 10)

22. Show that $\sin h^{-1}x=\ln\left(x+\sqrt{x^2+1}\right), -\infty<x<\infty$.

23 Does the series $\sum_{n=1}^{\infty}\frac{n+2^n}{n^2 2^n}$ converge. Justify.

24. Graph the lemniscate $r^2=4\cos 2\theta$. What symmetries do the curves have?

25. State term by term differentiation theorem. Express $f(x)=\frac{1}{1-x}$, $|x|<1$ as a power series. Use the theorem to find $f'(x)$ and $f''(x)$.

26. Find the area inside the smaller loop of the limacon $r=2\cos\theta+1$.

27. Find f_x, f_y and f_z, for the function $f(x,y,z)=x-\sqrt{y^2+z^2}$.

28. Find the linearization of the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ at $(1,0,0)$

IV. Essay Questions (Answer any two questions, weightage 2 x 4 = 8)

29 a. Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

b. Find the Taylor's series and Taylor polynomials generated by $f(x)=\cos x$ at $x=0$.

30. a. Does $\sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}+3\sqrt{n}}$ converge?

b. State Limit comparison test.

c. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{n\sqrt{n}}{2^n}$.

31. a. Find $\frac{\partial w}{\partial r}$ when $r=1$, $s=-1$ if $w=(x+y+z)^2$, $x=r-x$, $y=\cos(r+s)$, $z=\sin(r+s)$.

b. If $f(u,v,w)$ is differentiable and $u=x-y$, $v=y-z$ and $w=z-x$ show that $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=0$.