

FOURTH SEMESTER B.Sc MATHEMATICS EXAMINATION  
MATHEMATICS (CORE COURSE)  
MM4B04: MATHEMATICS  
Model Question Paper 2

Time :3 hours

weightage: 30

I. Objective Questions

(Answer all 12 questions. Weightage =  $12 \times \frac{1}{4} = 3$ )

1. The range of the exponential function  $y = e^x$  ( $x$  real) is ...
2. The Geometric Series  $\sum a r^{n-1}$  converges when
  - i)  $r = 1$
  - ii)  $r > 1$
  - iii)  $-1 < r < 1$
3. Let  $\sum a_n$  be a series with positive terms. The Ratio Test Fails when  
 $\text{Lim} (a_{n+1}/a_n) = \dots$
4. What is the domain of  $\tanh^{-1}(x)$  ?
5. The necessary condition for the convergence of an infinite series  $\sum a_n$  is ...
  - i)  $\lim a_n = 0$
  - ii)  $\lim a_n \neq 0$
  - iii)  $\lim a_n$  does not exist
6. The eccentricity  $e$  of a hyperbola is ...
  - i)  $e = 1$
  - ii)  $e < 1$
  - iii)  $e > 1$
7. The alternating series  $\sum (-1)^n / n$  converges to ...
8.  $x^2 + xy + y^2 - 1$  represents a
  - i) parabola
  - ii) ellipse
  - iii) hyperbola
9. The series  $\sum \sin(1/n)$  is ...
10. The polar equation  $r = a(1 + \cos \theta)$  represents the curve
  - i) circle
  - ii) parabola
  - iii) cardioid
11.  $\log_a x * \log_e a =$
12.  $\lim_{x \rightarrow 0} \sin x / x = ?$

II. Short Answer Questions

(Answer all Questions. Weightage :  $9 \times 1 = 9$ )

13.  $\lim_{x \rightarrow 0} (3^{\sin x} - 1) / x$
14. Find the eccentricity of the hyperbola  $9x^2 - 16y^2 = 144$
15. Find the sum of the series  $\sum 1/n(n+1)$
16. Define the Logarithmic Function  $\ln x$
17. If  $\sum a_n$  converges show that  $\lim a_n = 0$
18. Prove that  $f(x) = x - \ln x$  is increasing for  $x \geq 1$
19. Express  $a^x$  as an exponential function of  $x$

20. What is the path of P(x,y) moving in the XY plane given by the equation  $x = t^{1/2}$ ,  $y = t$ ,  $t \geq 0$

21. Find the derivative of  $7^x$

III. Short essay or Paragraph questions

(Answer any 5 questions. Weightage :  $5 \times 2 = 10$  )

22. Find the length of the cardioid  $r = 1 - \cos \theta$

23. Graph the Curve  $r^2 = \sin 2\theta$

24. Solve the initial value problem

$$d^2y/dx^2 = 2e^{-x}, \quad y(0) = 1, \quad y'(0) = 0$$

25. Order the following functions from slowest growing to

fastest growing as  $x \rightarrow \infty$

$$2^x, \quad x^2, \quad (\ln x)^x, \quad e^x$$

26. Check the convergence of the infinite series  $\sum \sin n / n^2$

27. Find  $\lim_{x \rightarrow 0} (1/\sin x - 1/x)$

28. Get an expression for  $\sinh^{-1}(x)$  using logarithms

IV. Essay questions

(Answer any two questions. Weightage:  $2 \times 4 = 8$  )

29. (a) State and prove Taylors Theorem

(b) Find the Taylor series and Taylor polynomials generated by

$$f(x) = \cos x \quad \text{at} \quad x=0$$

30. Check the convergence of the following infinite series

$$\text{i) } \sum (2^n + 5) / 3^n \quad \text{ii) } \sum 4^n n! n! / (2n)! \quad \text{iii) } \sum \ln(n) / n^{3/2}$$

31. Decide whether the equation  $x^2 + 2xy + y^2 + 2x - y + 2 = 0$  represent parabola, ellipse or hyperbola. Rotate the axis with the proper angle and reduce it to an equation without xy term. Find the foci, vertices and directrices of the conic.

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