FIRST SEMESTER B.Sc. DEGREE EXAMINATION
MATHEMATICS (CORE COURSE)
MM1B01: FOUNDATIONS OF MATHEMATICS
Model Question Paper 1

Time: 3 hrs. Maximum Weightage: 30

I. Objective type questions (Answer all 12 questions, weightage 12x¼ = 3)

Fill in the blanks

1. If A = {1, 3, 5} and B = {1, 2, 3}, then the symmetric difference $A \oplus B =$ ....
2. If n(A) = 10 and n(B) = 27, then $n(A \times B) =$ ....
3. If $(2x, x+y) = (8,6)$, then y = ...
4. If U is the universal set, A and B are subsets with $n(A) = 5$, $n(B) = 4$ and $n(A \cup B) = 7$, then $n(A \cap B) =$ ....
5. If $f : Z \to Z$ is the function $f(n) = n^2$, then $f^{-1}(\{-81, -25\}) =$ .... where Z is the set of integers.
6. If B is a subset of the universal set A, the characteristic function associated with B is ....
7. If $n(A) = 6$ and $n(B) = 8$, then the number of functions from A to B is ....
8. If $Q(x, y)$ denote the statement "*x* is the capital of *y*", then the truth value of the statement $Q(\text{Palakkad}, \text{Tamilnadu})$ is ....
9. Let $P(n)$ be the "$n^2 \geq 0$" where the domain is the set of all integers. the truth value of the quantification $\forall n P(n)$ is ....
10. The negation of the statement $\forall x (x^2 > x)$ is ....
11. In a graph G, the number of points and odd degree is ....
12. Let $G = (V, X)$ be the graph with $V = \{v_1, \cdots, v_p\}$, the sum of the i^{th} row of the adjacency matrix of *G* is ....

II. Short answer type questions (Answer all 9 questions, weightage 9x1=9)

13. If A = {1, 2, 3, 4}, is it true that $1 \in P(A)$, where $P(A)$ is the power set of A? Justify your answer.
14. If A, B and C are sets such that $A \cap C = B \cap C$, is it true that A = B? Justify your answer.
15. If A and B are two sets and $f : A \to B$ is a constant function, discuss the following cases.
    a) *f* is one-one
    b) *f* is onto
16. If S is any set and ***a*** is any element, is it true that S is equipotent with $S \times \{a\}$? Justify your answer.
17. Find all partitions of the set $A = \{1,2,3\}$.

18. Give the negation of the following proposition:

The summer in Chennai is hot and sunny

19. What is the conclusions drawn from the following premises and mention the rule of inference used?

Every student has an internet account

Raj has no internet account

20. Write the adjacency matrix of K_3, the complete graph on three points.
21. Argue the following statement if necessary with a counter example.

"Every connected graph is complete".

III. Short essay questions (Answer any 5 questions, weightage 5 x 2 = 10)

22. Among the 25 cars in a showroom, 15 have AC, 12 have radio, 11 have stereo, 5 have AC and stereo, 9 have AC and radio, 4 have radio and stereo and 3 have all the three. Find the number of cars with only one of the options.
23. If $I = \{1,2,3,....\}$ and for each $n \in I$, $A_n = \{n, 2n, 3n, \cdots\}$, find $\bigcap_{n \in I} A_n$.
24. If S and T are two sets $f : S \to T$ is a function, prove the following.

a) If A and B are subjects of S, $f(A \cap B) \subseteq f(A) \cap f(B)$.

b) If A is any subset of S, $A \subseteq f^{-1}(f(A))$.

25. Let A and B be two sets and $f : A \to B$. For $x, y \in A$, define $x \sim y$ if $f(x) = f(y)$. Show that $\sim$ is an equivalence relation on A.
26. If $P(x)$ is a polynomial of degree m, prove that $P(x) = O(x^m)$, where O stands for big O.
27. Prove that the following are equivalent statements

a) n is an odd integer.

b) n - 1 is an even integer.

28. Is there any graph with $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ as adjacency matrix? If so, draw the same.

IV. Essay type questions (answer any 2 questions, weightage 2x4 = 8)

29 (i) Prove that a function $f : A \to B$ is invertible if and only if it is one-one and onto.

(ii) P.T. an equivalence relation in a set A induces a partition of A and conversely.

30. (i) Prove that the sum of two odd numbers is even by direct proof.

(ii) Prove that if k is an integer and $3k + 2$ is even, then k is even, using a proof by contraposition and also a proof by contradiction

31. Prove the following

(i) If f is an isomorphism between the graphs $G_1 = (V_1, X_1)$ and $G_2 = (V_2, X_2)$ and if $v \in V_1$, prove that $\deg(v) = \deg(f(v))$.

(ii) Prove that a graph with k points is regular of degree k - 1 if and only if it is complete.

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Model Question Paper 2

Time: 3 hrs ***Maximum weightage : 30***

I. Objective type questions (Answer all 12 questions, wieghtage 12x¼ = 3)

Fill in the blanks

1. If A and B are sets with $A \subset B$, then the symmetric difference $A \oplus B$ = …………………
2. If U is the universal set, A and B are subsets with $n(A) = 5, n(B) = 4$ and $n\ (A \cup B) = 7$, then $n(A \cap B) =$ ....
3. If $(x^3, xy) = (-27, 24)$, then $y =$ ....
4. If A is any set and B = ϕ, then AxB = ...
5. If f : Z $\rightarrow$ Z is the function f (n) =n^3, then $f^{-1}(\{125, 27\}) =$ .... where Z is the set of integers (Fill in the blanks).
6. If $f : A \rightarrow B$ is on to then $f(A) =$ ....
7. If A = $\{1,2,3,4,\}$ and f is the function $\{(1, 1),(2,0),(3,1),(4,0),\}$ then f is the characteristic function of the subset B = ...
8. A compound proposition that is always false is called a ....
9. If the function $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = x + 1$ and $f(x) = x^2$ the $(g \circ f)(x) =$ ....
10. The negation of the statement $\exists x(x^2 = 2)$ is ....
11. The maximum degree of any point in a graph G with p points is ....
12. If A is the adjacency matrix of a simple graph, then the entries on the principal diagonal are ....

II. Short answer type questions (answer all 9 questions, weightage 9x1=9)

13. If S is any set, is it true that the empty set $\phi \subseteq S$? Justify your answer.
14. If U is the universal set, A and B are subsets with n $(U) = 10$, $n(A) = 7$, $n(B) = 5$, and n, $(A \cup B^C) = 8$ then n $n(A - B) =$ ....B^C is the complements of B in U.
15. Let A be any set and $P(A)$ be the powerset of A. DefineC<B, if $C \subseteq B$ for C, $B \in P(A)$. Is '<' an equivalence relation on P (A)? Justify your answer.
16. Are the sets A = {a,b,c,d,e,f} and $\{\alpha,\beta,\chi,\lambda,\mu\}$ have the same cardinality? Justify your answer.
17. If f:A $\rightarrow$B and g:B $\rightarrow$ C are functions, show that $g \circ f : A \rightarrow C$ is constant if either g or f is constant.
    Statement: If you have 90% marks, then you get A grade
18. Prove that $\neg(\neg p) = p$

19. Determine whether the following argument is correct or incorrect mentioning the rule:
All parrots like fruits. My pet bird is not parrot. Therefore, my pet bird does not like fruit.
20. Is there any 2-regular graph with 4 vertices and 5 edges? Justify your answer.
21. If G_1 is a 3-regular graph and G_2 is 6-regular graph, can they be isomorphic? Justify your answer.

III. Short Essay questions (answer any 5 questions, weightage 5x2=10)

22. If $\{ A_1\} i\in I$, is an indexed family of sets, prove De Morgan's law for this family.
23. Among the 120 students of a college, 40 take Mathematics, 50 takes English and 15 take both. Find the number of students take neither Mathematics nor English.
24. If U and V are two sets and $g:U \to V$ is a function, prove the following
a) If A and B are subset of U, $g(A\cup B)=g(A)\cup g(B)$.
b) For any subset B of V, $g\left(g^{-1}(B)\right)\subseteq B$
25. When do you say a function f grows faster than another function g? Prove that $f(n)=n!$ grows faster than the exponential function 2^n.
26. Prove that the square of an even number is an even number using
a) Direct proof
b) Proof by contradiction
27. What is an indirect proof? Using the contrapositive argument, prove that for an integer n, if 3n + 2 is odd, then n is odd.
28. If the first row of the incidence matrix of a graph is zero, what information can be deduced about that graph?

IV. Essay questions (Answer any 2 questions, weightage 2 x 4 = 8)

29. (a) If $\{ A_1\} i\in I$, is an indexed family of sets and B is any set prove that $B\cap(\cup A_i)=\cup_i(B\cap A_i)$

(b) Let U be a universal set $A\subseteq U$, $B\subseteq U$ and χ_A, χ_B are characteristic functions of A and B respectively. Prove that $\chi_{A\cup B}=\chi_A+\chi_B-\chi_{A\cap B}$
Also prove that if $p:U\to\{0,\ 1\}$, then there is a subset C of U such that $\chi_C = p$.

30. Prove the following

(a) If A,B,C,D are sets such that $A\approx C$ and $B\approx D$, prove that $AxB\approx CxD$

(b) Let $P(S)$ be the power set of a set S and for A, B $\in P(S)$ define $A\sim B$ if $A\approx B$. Prove that $\sim$ is an equivalence relation on $P(S)$.

(c) Prove that the propositions $p\to q$ and $\neg p \vee q$ are logically equivalent

31. (a) Discuss the following

(i) Konigsberg bridge problem

(ii) The four colour problem

(b) Prove that in any graph, the number of points of odd degree is even.

(c) Define indegree and outdegree. Prove that, in a digraph D, the sum of the indegrees of all vertices is equal to the sum of their outdegrees and each sum is equal to the number of arcs.