FIRST SEMESTER B.Sc. DEGREE EXAMINATION
MATHEMATICS (COMPLEMENTARY COURSE)
MM1C01: MATHEMATICS
Model Question Paper 1

Time: 3 hrs. Maximum Weightage: 30

I. Objective type questions.

(Answer **all 12** *questions. Weightage* **12 x ¼ = 3***)*

Fill in the blanks.

1. $\lim_{x\to 2}\left(4x^2-3\right)=$ ………………
2. If $2-x^2 \le g(x) \le 2\cos x$ for all *x*, then $\lim_{x\to 0} g(x)=$ ………………
3. $\lim_{x\to 0-}\frac{5}{2x}=$ ………………
4. The function $f(x)=\frac{\cos x}{x}$ is not continuous at $x=$ ………………
5. The curve $y=x^2-2x+1$ has a horizontal tangent at $x=$ ………………
6. The absolute maximum value of $f(x)=-x-4,\ -4\le x\le 1$, is at $x=$ ………………
7. If $f'(x)>0$ for every *x* in (a,b), then *f* is ……………… in (a,b)..
8. If $\lim_{x\to a+} f(x)=\pm\infty$, then the line ……………… is an asymptote of the curve $y=f(x)$.
9. If *f* is continuous and $\int_1^2 f(x)dx=-4$ and $\int_1^5 f(x)dx=6$, then $\int_2^5 f(x)dx=$ ………………
10. If *f* is integrable on $[a,b]$, then the average value of *f* on $[a,b]$ is $av(f)=$ ………………
11. The set $P=\{0,0.2,0.6,1,1.5,2\}$ is a partition of $[0,2]$. The norm of P is ___________.
12. The area bounded by the curve $y=f(x)$, *x*-axis and lines $x=a$, $x=b$ revolves round the *x*-axis. The volume of the solid generated is ………………

II. Short Answer type questions.

(*Answer* ***all 9*** *questions. Weightage* **9 x 1 = 9**)

13. If $f(x)=2x-4$ and $x_0=5$, $\varepsilon=0.2$ and $L=6$, find $\delta>0$, such that $0<|x-x_0|<\delta$ implies $|f(x)-L|<\varepsilon$.
14. Can you apply Rolle's theorem to the function.

$$f(x)=\begin{cases} x, 0\le x<1 \\ 0, x=1 \end{cases}$$

Justify your answer.

15. If x moves from left to right through the point $c=2$, is the graph of $f(x)=x^3-3x+2$ rising or falling? Justify your answer.

16. Use Sandwich theorem to find the asymptotes of $y=2+\frac{\sin x}{x}$.

17. Find the inflection point of the curve $f(x)=x^3-3x+3$.

18. Consider $f(x)=x^2-1$ in $[0,2]$. Partition the interval into four subintervals of equal length. Find the Riemann sum $\sum_{k=1}^{4} f(c_k)\Delta x_k$ where c_k is the left endpoint.

19. Find the average value of $f(x)=\frac{-x^2}{2}$ on $[0,3]$.

20. Find $\frac{dy}{dx}$ if $y=\int_1^{x^2}\cos t\,dt$.

21. Evaluate $\int_{-4}^{4}|x|\,dx$.

III. Short essay or Paragraph questions:

(*Answer any* **five** *questions. Weightage* **5 x 2 = 10**)

22. Draw the graph of the function.

$$f(x)=\begin{cases} x, & -1\le x<0 \\ 1, & x=0 \\ -x, & 0<x<1 \\ x-1, & 1\le x\le 2 \end{cases}$$

Which of the following statements about the function are true; and which are false?

a) $\lim_{x\to 0} f(x)$ exists

b) $\lim_{x\to 0} f(x)=0$

c) $\lim_{x\to 0} f(x)=1$

d) $\lim_{x\to 1} f(x)=1$

23. If a function is differentiable at $x=c$, prove that it is continuous at $x=c$. Is the converse true? Justify your answer.

24. Show that the function $f(x)=x^4+3x+1$, $-2\le x\le -1$ has exactly one zero in $[-2,-1]$.

25. If b, c, d are constants, for what value of b will the curve $y=x^3+bx^2+cx+d$ has a point of inflection at $x=1$. Justify your answer.

26. Express the solution of the initial value problem $\frac{dy}{dx}=\sec x$, $y(2)=3$, as an integral.

27. Find the area of the region enclosed by the curve $x=2y^2$, the line $y=3$ and the y-axis.

28. Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \le x \le 4$ about the *x*-axis.

IV. Essay questions.

(Answer any two questions. Weightage : 2 x 4 = 8)

29. Graph the function $f(x) = \begin{cases} 0, & x \le -1 \\ \frac{1}{x}, & 0 < |x| < 1 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$

Then discuss in complete detail, limits, one sided limits, continuity and one sided continuity of *f* at each of the points $x = -1$, 0 and 1. Are any of the discontinuities removable? Explain.

30. Graph the function $y = x^{5/3} - 5x^{2/3}$.

31.(a) Find the volume of the solid generated by revolving the region between the *y*-axis and the curve $xy = 2$, $1 \le y \le 4$ about the *y*-axis.

(b) Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

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Model Question Paper 2

Time: 3 hrs. Maximum Weightage: 30

I. Objective type questions:

(*Answer* **all 12** *questions. Weightage* **12 x ¼ = 3**)

Fill in the blanks.

1. $\lim_{x\to -7}(2x+5) =$ ……………..
2. A function *f* is continuous at an interior point $x = c$ of its domain if $\lim_{x\to c} f(x) =$ ……………..
3. The function $f(x) = \frac{1}{x-2} - 3x$ is not continuous at $x =$ ……………..
4. The slope of the curve $y = \frac{1}{x}$ at $x = 1$ equals ……………..
5. If $\lim_{x\to 0} f(x) = \frac{1}{2}$, then $\lim_{x\to 0}\frac{f(x)\cos x}{x-1} =$ ……………..
6. The absolute minimum value of the function $f(x) = x^2 - 1$, $-1 \le x \le 2$ is ……………...
7. If $y'' > 0$ on the interval I, then the graph of $y = f(x)$ over I is ……………..
8. The horizontal asymptote of the curve $y = \frac{1}{x}$ is ……………..
9. If *f* and *g* are continuous and that $\int_7^9 f(x)\,dx = 5$, $\int_7^9 g(x)\,dx = 4$, then $\int_7^9 \left(2f(x) + h(x)\right)dx =$ ……………..
10. The norm of the partition $P = \{0, 1.2, 1.5, 2.3, 2.6, 3\}$ is ……………..
11. $\frac{d}{dx}\left(\int_0^x \frac{1}{1+t^2 dt}\right) =$ ……………..
12. If *f* is smooth on $[a,b]$, the length of the curve $y = f(x)$ from *a* to *b* is $L =$ ……………..

II. Short answer type questions:

(*Answer* **all nine** *questions. Weightage* **9 x 1 = 9**)

13. Find the value of $\delta > 0$ such that for all *x*, $0 < |x - x_2| < \delta$ implies $1 < x < 7$.
14. Find the value of *c* that satisfy the mean value theorem for the function $f(x) = x^2 + 2x - 1$ on $[0,1]$.

15. Sketch the graph of a differentiable function $y = f(x)$ through $(1,1)$ if $f'(1) = 0$ and $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$.

16. Show that $\lim_{x \to \infty} \frac{1}{x} = 0$.

17. Find the linearization of $f(x) = x^2$ at $x = 1$.

18. Consider the function $f(x) = x^2 - 1$ on $[0,2]$. Partition the interval into four subintervals of equal length. Find the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$ where c_k is the left end point.

19. State the mean value theorem for definite integrals.

20. Find $\frac{dy}{dx}$ if $y = \int_0^{\sqrt{x}} \cos t\,dt$.

21. Find the average value of $f(x) = 3x^2 - 3$ on $[0,1]$.

III. Short essay.

(*Answer any* **five** *questions. Weightage* **5 x 2 = 10**)

22. Graph the function

$$f(x) = \begin{cases} x+2, & x \le 2 \\ -x+2, & -2 < x \le -1 \\ -1, & -1 \le x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

Find the limits or explain why they do not exist?

a) $\lim_{x \to -2} f(x)$ b) $\lim_{x \to -1} f(x)$

c) $\lim_{x \to 0} f(x)$

23. Define $\lim_{x \to x_0} f(x) = L$, where *f* is a function defined on an open interval containing x_0. For $\in > 0$ find $\delta > 0$ such that when $0 < |x-3| < \delta$ implies $|f(x)+3| < \in$ where $f(x) = 3 - 2x$.

24. For what values of *a*, *m* and *b* does the function.

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + 9 & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the mean value theorem

25. Find the extreme points and the points of inflection if any, for the curve $y = x^3 - 4x + 3$.

26. Find $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

27. Find the area of the region between the curve $y = 4 - x^2$, $0 \le x \le 3$, and the x-axis.

28. The region between the curve $y = 2\sqrt{x}$, $0 \le x \le 2$, and the x-axis. Is revolved about the x-axis. Find the volume of the solid generated.

IV. Essay questions.

(*Answer* **any 2** *questions. Weightage* **2 x 4 = 8**)

29. Use the following information to graph the function f over the interval $[-2,5]$.

(a) The graph is made of closed line segments joined end to end.

(b) The graph starts at $(-2,3)$.

(c) The derivative of f is the function given by

$$f'(x) = \begin{cases} -2 & -2 \le x < 0 \\ 0 & 0 < x < 1 \\ 1 & 1 < x < 3 \\ -1 & 3 < x \le 5 \end{cases}$$

Repeat assuming the graph starts at $(-2,0)$ instead of $(-2,3)$.

30. Let $f(x)$ be a continous function. Express

$$\lim_{x\to\infty} \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{n}{n}\right)\right]$$

as definite integral. Use the result to evaluate

(a) $\lim_{n\to\infty} \frac{1}{n^2}\left[2+4+6+\cdots+2n\right]$

(b) $\lim_{n\to\infty} \frac{1}{n^{16}}\left[1^{15}+2^{15}+\cdots+n^{15}\right]$

(c) $\lim_{n\to\infty} \frac{1}{n}\left(\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \cdots + \frac{n\pi}{n}\right)$

or

31.(i) Find the volume of the solid generated by revolving the region bounded on the left by circle $x^2 + y^2 = 3$ on the right by the line $x = \sqrt{3}$ and above by the line $y = \sqrt{3}$.

or

(ii) (a) Find the length of the curve $x = \frac{y^{\frac{3}{2}}}{3} - y^{\frac{1}{2}}$ from y = 1 to y = 9.

(b) Find the volume of the solid generated by revolving the region between the parabola $x = y^2+1$ and the line x = 3 about the line x=3.