SECOND SEMESTER B.Sc. DEGREE EXAMINATION
MATHEMATICS (COMPLEMENTARY COURSE)

MM2C02: MATHEMATICS

Model Question Paper 1

Time: 3 hrs **Maximum weightage : 30**

I. Objective type questions (Answer all questions, Weightage 12x¼ = 3)

1. The range of the function $y = \cos hx$ is ………………..
2. An example of an increasing sequence which is bounded above is ………………
3. The derivative of $\sec h2x$ with respect to x is ……………..
4. A convergent subsequence of the sequence $\{(-1)^n\}$ is ……………….
5. The n[th] term of the sequence 0,1,2,2,3,3,4,4, is ……………….
6. $\lim_{n\to\infty} a_n$ where $a_n = \sqrt{\dfrac{3n}{n+3}}$ is
7. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n$ = ……………
8. The least upper bound of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots, \frac{n}{n+1}, \cdots$ is …………
9. The polar equation of the cartesian equation $x = 7$ is…………..
10. The radius of the circle $r = 6 \sin \theta$ is …………….
11. An example of a bounded region in the plane is ………………
12. Cartesian coordinates of the point (1,0) (given in polar coordinates) is …………………

II. Short answer type questions (Answer all nine questions. Weightage 9 x 1 = 9).

13. $\int_{-\ln 4}^{-\ln 2} 2e^x \cos hx\, dx$

14. Evaluate $\int_{-1}^{1} \frac{dx}{x^{2/3}}$.

15 Fin $\lim_{n\to\infty} a_n$ where $a_n = \frac{\sin n}{n}$.

16 Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$.

17 Examine the convergence of the series $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n$. If it converges find its sum.

18 Prove or disprove $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ converges.

19 State Rearrangement theorem for absolutely convergent series.

20 Find an equation of the hyperbola with eccentricity $\frac{3}{2}$ and directix $x=2$.

21. Find the gradient of $f(x,y)=y-x$ at $(2,1)$.

III. Short essay questions (Answer any five questions, weightage 5 x 2 = 10)

22. Show that $\sin h^{-1}x=\ln\left(x+\sqrt{x^2+1}\right),-\infty<x<\infty$.

23 Does the series $\sum_{n=1}^{\infty}\frac{n+2^n}{n^2 2^n}$ converge. Justify.

24. Graph the lemniscate $r^2=4\cos 2\theta$. What symmetries do the curves have?

25. State term by term differentiation theorem. Express $f(x)=\frac{1}{1-x}$, $|x|<1$ as a power series. Use the theorem to find $f'(x)$ and $f''(x)$.

26. Find the area inside the smaller loop of the limacon $r=2\cos\theta+1$.

27. Find f_x, f_y and f_z, for the function $f(x,y,z)=x-\sqrt{y^2+z^2}$.

28. Find the linearization of the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ at $(1,0,0)$

IV. Essay Questions (Answer any two questions, weightage 2 x 4 = 8)

29 a. Evaluate $\int_0^3\frac{dx}{(x-1)^{2/3}}$.

b. Find the Taylor's series and Taylor polynomials generated by $f(x)=\cos x$ at $x=0$.

30. a. Does $\sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}+3\sqrt{n}}$ converge?

b. State Limit comparison test.

c. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{n\sqrt{n}}{2^n}$.

31. a. Find $\frac{\partial w}{\partial r}$ when $r=1$, $s=-1$ if $w=(x+y+z)^2$, $x=r-x$, $y=\cos(r+s)$, $z=\sin(r+s)$.

b. If $f(u,v,w)$ is differentiable and $u = x - y$, $v = y - z$ and $w = z - x$ show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$.

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Model Question Paper 2

Time: 3 hrs. Maximum Weightage: 30

I. Objective type questions (Answer all 12 questions, wieghtage 12 x ¼ = 3)

Fill in the blanks

1. The value of k such that the integral $\int_1^\infty x^{-k}dx$ diverges is ....
2. An example of a divergent sequence ....
3. A convergent subsequence of the sequence $\left\{\left(\sin n\frac{\pi}{2}\right)\right\}$......................
4. The n^{th} term of the sequence $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \ldots$ is ....
5. The series $\sum_{n=1}^{\infty} n^2$ is ....
6. The derivative of $y = \text{cosec}\, h(3x)$ w.r.t. to x is ....
7. The value of $\sin h^{-1}1$ using logarithm is ....
8. The range of $\cot h^{-1}x$ is ....
9. The cylindrical coordinates of $(0,1,0)$ (in Cartesian coordinate) is ....
10. In polar coordinates the region $\{(x, y) : x \geq 0, y \geq 0\}$ (in cartesian coordinates) is ....
11. The graph of the polar equation $r = 2$ is a ....
12. If $z = e^{xy}.\ln y$ then $\frac{\partial z}{\partial x} =$....................

II. Short answer type questions (Answer all 9 questions, weightage 9x1 = 9)

13. Find $\int_0^1 \frac{dx}{\sqrt{x}}$.
14. Show that $\lim_{n \to \infty} \frac{\ln}{n} = 0$.
15. Check the convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$
16. Show that the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges.
17. State the series multiplication theorem for power series.
18. Define boundary point. Give an example.
19. Define a function $f(x, y)$ is continuous at the point (x_0, y_0).
20. Find the gradient of $f(x, y) = y - x^2$ at $(-1, 0)$.

21. Find $\frac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$; $t = \pi$.

III. Short Essay questions (Answer any 5 questions, weightage 5 x 2 = 10)

22. Investigate the convergence of $\int_1^\infty e^{-x^2}\,dx$.

23. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n^2}\right)$.

24. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$. Where, if anywhere, does the series converge to $\frac{1}{x}$?

25. Graph the curve $r^2 = \sin 2\theta$

26. Find f_x, f_y and f_z for the function $f(x,y,z) = \left(x^2 + y^2 + z^2\right)^{-\frac{1}{2}}$.

27. Find the linearizations of the function $f(x,y,z) = e^x + \cos(y+z)$ at $(0,0,0)$.

28. Find the equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point $(-2,1)$.

IV. Essay questions (Answer any 2 questions, weightage 2x4 = 8)

29. a. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} n^{-n}$.

b. Show that the alteration harmonic series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ converges.

c. Define absolute convergence and conditional convergence of a series what is the relation between these two?

30.(i) a. Find the points of intersection of the curves $r = 1 + \cos\theta$, $r^2 = 4\cos\theta$.

b. Graph the curve $r = \frac{8}{4 + \cos\theta}$.

c. Find the area of the region shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$.

or

31. a. Find $\frac{\partial w}{\partial v}$ when $u = 0$, $v = 0$ if $w = x^2 + \left(\frac{y}{x}\right)$, $x = u - 2v + 1$, $y = 2u + v - 2$.

b. Find the directional derivative of $f(x,y) = 2xy - 3y^2$ at $(5,5)$ in the direction $\vec{a} = 4\vec{i} + 3\vec{j}$.

c. Find the equation for the tangent plane and the normal line for the surface $x^2 + y^2 + z^2 = 3$ at $(1,1)$.